# **Energy scale of the electron-boson spectral function and superconductivity in NpPd<sub>5</sub>Al<sub>2</sub>**

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The energy scale  $\Omega_0$  of the electron-boson spectral function in the heavy-fermion, *d*-wave superconductor  $NpPd<sub>5</sub>Al<sub>2</sub>$  is predicted on the basis of Eliashberg theory calculations. Assuming a spectral function shape typical for antiferromagnetic spin fluctuations, and imposing constraints provided by the experimental values for the critical temperature and the low-temperature energy gap, one obtains values of  $\Omega_0$  of about 2–2.5 meV, slightly dependent from the strength of the Coulomb pseudopotential. These values are in excellent agreement with the characteristic magnetic fluctuations energy estimated from NMR measurements of the nuclear-spinlattice relaxation time at the Al site. The calculated temperature dependence of the upper critical field, the local spin susceptibility, and the nuclear-spin-lattice relaxation rate is also in good agreement with available experimental data, showing that a coherent description of the superconducting state can be obtained assuming that the electron pairing in  $NpPd_5Al_2$  is mediated by antiferromagnetic fluctuations. We finally report predictions for the London penetration depth, the energy dependence of the tunneling differential conductance at different temperatures, and the temperature dependence of the energy gap.

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## **I. INTRODUCTION**

The properties of actinide compounds are often unusual. In systems with well-localized 5*f* electrons, in presence of unquenched orbital degrees of freedom and strong spin-orbit coupling, higher-order electromagnetic multipole interactions affect the ground state and the low-energy dynamics.<sup>1</sup> On the contrary, where the 5*f* states form narrow bands or are significantly hybridized with ligand- or conductionelectron states, the coexistence of atomic and metallic electron behavior can lead to distinct physics not described by the standard Landau-Fermi-liquid theory.<sup>2</sup>

Particularly interesting is the case of Pu- and Np-based heavy-fermion intermetallic superconductors, providing evidence for unconventional Cooper pairing not mediated by electron-phonon interactions.<sup>3</sup> A striking feature of these compounds is the value of the critical temperature,  $T_c$  $= 18.5$  K for PuCoGa<sub>5</sub> (Ref. [4](#page-5-3)) and 4.9 K for NpPd<sub>5</sub>Al<sub>2</sub>,<sup>[14](#page-5-4)</sup> which is one order of magnitude higher than for typical *f*-electron superconductors. Several characteristics of the superconducting and normal states suggest that spin fluctuations may have an important role in the stabilization of the ground state of  $PuCoGa<sub>5</sub>$ , although definite evidence on the nature of the mediating bosons is still missing. $6-8$ 

 $NpPd<sub>5</sub>Al<sub>2</sub>$  is the only known superconducting Np compound. Following its discovery by Aoki *et al.*, [5](#page-5-7) extensive studies on its physical properties have been reported. $9-12$ NpPd<sub>5</sub>Al<sub>2</sub> has a body-centered tetragonal crystallographic structure, with space group *I*4/*mmm* and lattice parameters  $a=4.1010$  Å and  $c=14.6851$  Å at room temperature. The magnetic susceptibility in the normal phase follows the Curie-Weiss behavior and is strongly anisotropic. When the external magnetic field  $\bf{B}$  is applied along the  $[100]$  direc-

tion, the effective paramagnetic moment  $\mu_{eff}$  has a value of 3.2  $\mu_B/Np$  and the Curie-Weiss temperature is  $\theta_p = -42$  K; for **B** $\parallel$ [001],  $\mu_{eff}$ =3.06  $\mu_B$ /Np and  $\theta_p$ =-139 K. Electricalresistivity and specific-heat measurements reveal a non-Fermi-liquid behavior of the normal phase, and the recovery of Fermi-liquid behavior in presence of magnetic fields larger than the upper critical field  $H_{c2}(0)$ .<sup>[12](#page-5-9)[,13](#page-5-10)</sup> With increasing pressure, the superconducting transition temperature  $T_c$ decreases and becomes zero above 5.7 GPa, indicating proximity to a quantum critical point.<sup>13</sup> The magnetic contribution to the electrical resistivity suggests the presence of Kondo interactions in a crystal-field potential, and the magnitude and overall temperature dependence of the thermoelectric power are typical of dense Kondo systems[.12](#page-5-9) Mössbauer spectroscopy experiments indicates that the Np ions are nearly trivalent, and the negative sign of the Seebeck coefficient indicates that charge and heat transport are dominated by electrons, $^{12}$  in agreement with band-structure calculations[.14](#page-5-4)

The temperature dependence of the electronic specific heat  $C_e$  suggests the presence of a large density of states at the Fermi level. In the temperature range between 1 and 3 K, the Sommerfeld coefficient  $\gamma = C_e / T$  reaches a value of 325 mJ/mol  $K^2$  when a magnetic field of 14 T is applied.<sup>12</sup> The specific-heat jump at  $T_c$  is  $\Delta C_e / \gamma T_c = 2.33$ . Below  $T_c$ ,  $C_e$ follows a  $T^2$  dependence characteristic of strong coupling and suggesting the presence of line nodes in the energy gap  $\Delta(T)$ . The nuclear-spin-lattice relaxation rate  $1/T_1$ , deduced from  $37$ Al NMR spectra,<sup>11</sup> shows no coherence peak and varies, below  $T_c$ , as  $T^3$ . Assuming *d*-wave gap symmetry with line nodes, the observed temperature dependence gives<sup>11</sup> 2 $\Delta(0)/k_B T_c \approx 6.4$ . Above  $T_c$ ,  $1/T_1 T$  decreases with

increasing temperature, a behavior compatible with the presence of magnetic fluctuations.<sup>11</sup>

Electronic band-structure calculations<sup>14</sup> show the presence of a band with 5*f* character, having a large Fermi surface and a narrow bandwidth. This result supports the hypothesis that superconductivity in NpPd<sub>5</sub>Al<sub>2</sub> involves 5*f* electrons. The possibility that nearly antiferromagnetic spin fluctuations provide the mediating bosons leading to *d*-wave pairing and superconductivity has been investigated by Tanaka and Hanzawa[.15](#page-5-12) These authors calculate the dynamical susceptibility using a single-band Hubbard model with tight-binding dispersion and the fluctuation exchange approximation to study the effect of the on-site Coulomb repulsion[.15](#page-5-12) Their results show that the features in the  $NpPd<sub>5</sub>Al<sub>2</sub>$  band structure enhance strong incommensurate magnetic fluctuations favoring superconductivity with  $d_{x^2-y^2}$ symmetry.<sup>16</sup>

Here, we report the results of *d*-wave Eliashberg theory<sup>17[–19](#page-5-15)</sup> calculations performed in order to determine the energy scale of the mediating bosons spectral function that would be compatible with observed experimental data. We show that a spectral function centered around 2–2.5 meV is required to account for a critical temperature value  $T_c$  $\approx$  5 K and an energy gap  $\Delta(0)$ =1.35 meV. These values compare very favorably with the characteristic magnetic fluctuations energy that can be deduced from 37Al NMR measurements of the spin-lattice relaxation rate at the Al site.<sup>[20](#page-5-16)</sup> The solutions of the Eliashberg equations are also used to calculate the temperature dependence of several physical observable, namely, the energy gap, the upper critical field, the London penetration depth, the local spin susceptibility, the nuclear-spin-lattice relaxation rate, and the energy dependence of the tunneling conductance. The good agreement between calculations and available experimental data indicates that the present model gives a coherent description of the superconducting state in  $NpPd<sub>5</sub>Al<sub>2</sub>$  if the electron pairing is mediated by magnetic fluctuations.

#### **II. THEORETICAL MODEL**

In the imaginary-frequency-axis formulation, the *d*-wave one-band Eliashberg equations for the renormalization functions  $Z(i\omega_n, \phi)$  and the gap  $\Delta(i\omega_n, \phi)$  are given by<sup>21</sup>

$$
\omega_n Z(i\omega_n, \phi) = \omega_n + \pi T \sum_m \int_0^{2\pi} \frac{d\phi'}{2\pi} \Lambda(i\omega_n - i\omega_m, \phi, \phi')
$$

$$
\times N_Z(i\omega_m, \phi') + \Gamma_N \frac{N_Z(i\omega_n)}{c^2 + N_Z(i\omega_n)^2},
$$
(1)

$$
Z(i\omega_n, \phi) \Delta(i\omega_n, \phi)
$$
  
=  $\pi T \sum_m \int_0^{2\pi} \frac{d\phi'}{2\pi} [\Lambda(i\omega_n - i\omega_m, \phi, \phi')$   
 $- \mu^*(\phi, \phi', \omega_c) \vartheta(\omega_c - |\omega_m|)] N_{\Delta}(i\omega_m, \phi'),$  (2)

where  $\vartheta(\omega_c - |\omega_m|)$  is the Heaviside function,  $\omega_c$  is an electronic cut-off energy for the Coulomb pseudopotential  $\mu^*$ ,  $N_Z(i\omega_m, \phi) = \omega_m / \sqrt{\omega_m^2 + \Delta^2(i\omega_m, \phi)}$ ,  $N_Z(i\omega_n)$  is the

average over  $\phi$  of  $N_Z(i\omega_n, \phi)$ ,  $N_{\Delta}(i\omega_m,\phi)$  $=\Delta(i\omega_m, \phi) / \sqrt{\omega_m^2 + \Delta^2(i\omega_m, \phi)}$ , and

$$
\Lambda(i\omega_n - i\omega_m, \phi, \phi') = 2 \int_0^{+\infty} d\Omega \frac{\Omega \alpha^2 F(\Omega, \phi, \phi')}{(\omega_n - \omega_m)^2 + \Omega^2}.
$$
 (3)

 $\Omega$  being the mediating boson frequency and  $\alpha^2 F(\Omega, \phi, \phi')$  the electron-boson spectral function.

The parameter  $\Gamma_N$  is proportional to the concentration of impurities or to disorder, and *c* is a parameter related to the electron phase shift for scattering off an impurity  $(c = \infty$  with a constant value of  $\Gamma_N/c^2$  corresponds to the Born limit, whereas the unitary limit is obtained when  $c = 0$ .<sup>[22](#page-5-18)</sup> The *n*th Matsubara frequency is defined as  $i\omega_n = i\pi T(2n-1)$ , with *n*  $=0, \pm 1, \pm 2...$ , and *T* is the temperature.

In the following, calculations will be performed in the clean limit, that is assuming  $\Gamma_N=0$ , unless stated otherwise. As usual, we assume that  $\alpha^2(\Omega) F(\Omega, \phi, \phi')$  and  $\mu^*(\phi, \phi')$ contain at the lowest order separate *s*- and *d*-wave contributions, that is

$$
\alpha^2 F(\Omega, \phi, \phi') = \alpha^2 F_s(\Omega) + \alpha^2 F_d(\Omega) \sqrt{2} \cos(2\phi) \sqrt{2} \cos(2\phi'),
$$
\n(4)

-

$$
u^*(\phi, \phi') = \mu_s^* + \mu_d^*(\Omega) \sqrt{2} \cos(2\phi) \sqrt{2} \cos(2\phi').
$$
 (5)

We adopt a simplified model, and we search for solutions of the Eliashberg equations with a pure *d*-wave symmetry gap function,  $\Delta(\omega, \phi') = \Delta_d(\omega) \cos(2\phi')$ , and a pure *s*-wave renormalization function,  $Z(\omega, \phi') = Z_s(\omega)$ . We note that, assuming *d*-wave symmetry for the gap function, the parameter  $\mu_s^*$  does not enter into the two relevant Eliashberg equations. Therefore, although it is almost certainly large, it does not influence the solution. For simplicity, we finally assume that the normal density of states is constant around the Fermi level and that  $\alpha^2 F_s(\Omega) = \alpha^2 F_d(\Omega)$ .

The antiferromagnetic spin fluctuations spectral function for a uniform electron gas can be calculated in a T-matrix approximation,<sup>23</sup> and is proportional to a function  $P(\Omega)$  $=\Omega \Gamma/(\Omega^2 + \Gamma^2)$ , peaked at an energy  $\Omega_0$  of the order of  $\Gamma$ . For numerical calculation convenience, we approximate the corresponding Eliashberg spectral function as the difference of two Lorentzian curves,  $\alpha^2 F(\Omega) \propto [L(\Omega - \Omega_0, Y) - L(\Omega_0)]$  $+\Omega_0$ , Y)], where  $L(\Omega \pm \Omega_0, Y) = [(\Omega \pm \Omega_0)^2 + (Y)^2]^{-1}$ ,  $\Omega_0$  is the energy of the peak, and  $Y$  determines its half width at half maximum. We constrain the electronic cut-off energy to  $\omega_c = 24\Omega_0$  and the maximum quasiparticle energy to  $\omega_{\text{max}}$  $= 32\Omega_0$ .

## **III. CALCULATION OF THE MEDIATING BOSONS ENERGY SCALE**

With the above assumptions, four free parameters remain, namely,  $\lambda_s$ ,  $\Omega_0$ ,  $\mu_d^*$ , and Y. On the other hand, the experi-ments provide two constraints,<sup>5[,11](#page-5-11)</sup>  $T_c = 5$  K and  $\Delta(0)$ = 1.35 meV. We then proceed by assigning a value to  $\mu_d^*$  and Y, we calculate the  $\lambda_s(\Omega_0)$  curve corresponding to a  $T_c$  $= 5$  K by solving the Eliashberg equations, and for each value of  $\Omega_0$  we use the corresponding  $\lambda_s$  to calculate the

<span id="page-2-0"></span>

FIG. 1. (Color online) (Left axis) Gap values  $\Delta_d$  calculated at  $T=0.25$  K assuming a Coulomb pseudopotential  $\mu_d^* = 0$ , and an Eliashberg spectral function centered at an energy  $\Omega_0$  and having full width at half maximum  $Y=1$  meV (red, open circles) or Y  $= 0.25$  meV (blue, open squares). The horizontal dashed line represents the experimental value of  $\Delta_d$ . (Right axis) Electron-boson coupling constant  $\lambda_s$  as a function of  $\Omega_0$ , for  $T_c = 5$  K,  $\mu_d^* = 0$ , and  $Y=1$  meV (red, closed circles) or  $Y=0.25$  meV (blue, closed squares). The inset shows the electron-boson spectral function in the two cases.

low-temperature gap,  $\Delta_d$ , via Padè approximants.<sup>24,[25](#page-5-21)</sup> The value of  $\Omega_0$  compatible with the experimental determination of  $\Delta_d$  is then immediately determined. The calculations are repeated for different values of  $\mu_d^*$  and Y, to assess the influence of these parameters on the estimated energy scale of the electron-boson spectral function. The use of the Padè approximants technique for the calculation of  $\Delta_d$  is preferable when dealing with strong-coupling superconductors because the value of  $\Delta_d(i\omega_{n=0})$  obtained by solving the imaginary-axis Eliashberg equations can in these cases differ considerably from the value of  $\Delta_d$  provided by the realfrequency-axis Eliashberg equations[.26](#page-5-22)

Figure [1](#page-2-0) shows the results of the procedure described above for  $\mu_d^* = 0$ , and  $Y = 1$  meV or 0.25 meV. For Y =1 meV, we obtain  $\Omega_0 = 2$  meV and  $\lambda_s = 2.48$ ; the value of  $\Omega_0$  does not change significantly if Y is reduced to 0.25 meV, whereas  $\lambda_s$  decreases to 2.24. Indeed, the critical temperature  $T_c$  is an increasing function of both  $\lambda_s$  and of  $\Omega_{log}$ , a characteristic energy of the electron-boson spectral function,  $\Omega_{log} = \exp\left[\frac{2 \int_0^{+\infty} d\Omega \ln(\Omega) \alpha^2 F_s(\Omega) / \Omega \right] / \lambda_s \right]$ .<sup>[18](#page-5-23)</sup> With decreasing Y,  $\Omega_{log}$  increases toward  $\Omega_0$  ( $\Omega_{log}$ =1.29 meV for Y  $= 1$  meV and  $\Omega_{log} = 1.69$  meV for  $\Upsilon = 0.25$  meV), and hence  $\lambda_s$  must decrease to keep  $T_c$  constant.

The effect of the Coulomb pseudopotential is illustrated in Fig. [2,](#page-2-1) where the  $\Omega_0$  dependence of  $\lambda_s$  and  $\Delta_d$  is shown for different values of  $\mu_d^*$ , at fixed  $T_c = 5$  K. If  $\mu_d^*$  increases from 0 to 0.2,  $\Omega_0$  increases up to about 2.5 meV, and  $\lambda_s$  up to about 3.1. We therefore conclude that magnetically mediated superconductivity in  $NpPd<sub>5</sub>Al<sub>2</sub>$  requires spin fluctuations characterized by a dominating energy scale between 2 and 2.5 meV.

## **IV. ESTIMATION OF** *Ω***<sup>0</sup> FROM NMR SPIN-LATTICE RELAXATION TIME**

An experimental estimation of  $\Omega_0$  can be obtained from NMR measurements of  $1/T_1$ , the spin-lattice relaxation rate

<span id="page-2-1"></span>

FIG. 2. (Color online) Gap values  $\Delta_d$  (upper panel) and electron-boson coupling constant  $\lambda_s$  (lower panel) calculated at *T*  $= 0.25$  K assuming  $T_c = 5$  K and a Lorentzian-shaped boson spectral function with center at  $\Omega_0$  and full width at half maximum Y =1 meV: (red, open circles)  $\mu_d^*$ =0; (blue, closed circles)  $\mu_d^*$ =0.1; (black squares)  $\mu_d^* = 0.2$ . The horizontal dashed line gives the experimental value of  $\Delta_d$ .

at the Al site, in the normal state of  $NpPd_5Al_2$ .<sup>[20](#page-5-16)</sup> Generally,  $1/T_1$  is related to the dynamical magnetic susceptibility Im  $\chi(\mathbf{q},\Omega)$ ,<sup>[27](#page-5-24)</sup>

$$
\frac{1}{T_1} = 2\gamma_N^2 T \sum_q A^2(q) \frac{\operatorname{Im} \chi(\mathbf{q}, \omega_n)}{\omega_n},\tag{6}
$$

where  $\gamma_N$ ,  $A(q)$ , and  $\omega_n$  are the nuclear gyromagnetic ratio, the hyperfine coupling constant, and the NMR measurement frequency  $(\sim 110$  MHz in the present case), respectively. Since the hyperfine form factor  $f^2(q)=1$  at the Al site of NpPd<sub>5</sub>Al<sub>2</sub>, and  $A(q) \approx A(0) f(q)$ <sup>[20](#page-5-16)</sup>

$$
\frac{1}{T_1} = 2\gamma_N^2 A^2(0) T \sum_q \frac{\operatorname{Im} \chi(\mathbf{q}, \omega_n)}{\omega_n}.
$$
 (7)

<span id="page-2-2"></span>The value of  $A(0)$  has been determined by Chudo *et al.*<sup>[20](#page-5-16)</sup> from the Knight shift versus static susceptibility plot. On the assumption that Im  $\chi(\mathbf{q}, \Omega)$  can be described in the Lorentzian form with the characteristic magnetic fluctuation energy  $\Gamma$ , one has

$$
\frac{\operatorname{Im}\,\chi(\mathbf{q},\Omega)}{\Omega} \sim \frac{\chi(\mathbf{q})}{\Gamma_q}.\tag{8}
$$

<span id="page-2-3"></span>The *q*-averaged value of  $\Gamma$  can be determined from Eqs. ([7](#page-2-2)) and ([8](#page-2-3)) combined with the strong correlation approximation  $2\pi\chi(q)\Gamma(q) \approx 1,^{28}$ 

<span id="page-3-0"></span>

FIG. 3. (Color online) Temperature dependence of the characteristic magnetic fluctuation energy  $\Gamma$  in NpPd<sub>5</sub>Al<sub>2</sub> estimated from the spin-lattice relaxation time NMR measurements at the Al site (Ref. [20](#page-5-16)). The inset shows the assumed  $\Omega$  dependence of the dynamical susceptibility. The value of  $\Gamma$  corresponds to the energy of the maximum of the dynamical susceptibility.

$$
\Gamma^2 = \frac{1}{\pi} \gamma_N^2 A^2(0) T_1 T.
$$
 (9)

Although it is *q* averaged, the above expression for  $\Gamma$  corresponds in the present case to that around the antiferromagnetic wave vector *Q*, for Im  $\chi(\mathbf{q}, \Omega)$  is enhanced around *Q*.<sup>[15](#page-5-12)</sup>

Experiments have been performed on crystals aligned with the *a* or *c* crystallographic axis parallel to the external magnetic field and the isotropic part of  $\Gamma$  has been obtained as  $\Gamma = (2\Gamma_a + \Gamma_c)/3$ . The results are shown in Fig. [3.](#page-3-0) The parameter  $\Gamma$ , whose value increases from about 1 to about 2 meV as the temperature increases up to 300 K, corresponds to the energy of the maximum of Im  $\chi(\mathbf{q},\Omega)$ , shown in the inset of Fig. [3,](#page-3-0) and can therefore be compared with the predicted value of  $\Omega_0$ . The results obtained demonstrate that the characteristic magnetic fluctuation energy in  $NpPd<sub>5</sub>Al<sub>2</sub>$  is of the correct order of magnitude to induce superconductivity below  $T_c \approx 5$  K.

### **V. OTHER OBSERVABLES**

#### **A. Upper critical field and penetration depth**

The upper critical field  $H_{c2}$  can be calculated as a function of temperature by solving the linearized Eliashberg equations in presence of a magnetic field. $29,30$  $29,30$  In the clean limit (negligible impurity scattering),

$$
\omega_n Z_s(i\omega_n) = \omega_n + \pi T \sum_m \frac{1}{2\pi} \int_0^{2\pi} d\phi'
$$
  
 
$$
\times \Lambda(i\omega_n - i\omega_m, \phi, \phi') sign(\omega_m), \qquad (10)
$$

$$
Z_s(i\omega_n)\Delta_d(i\omega_n,\phi) = \pi T \sum_m \frac{1}{2\pi} \int_0^{2\pi} d\phi' [\Lambda(i\omega_n - i\omega_m, \phi, \phi') - \mu_d^*(\omega_c, \phi, \phi')] \theta(\omega_c - |\omega_m|)
$$
  
 
$$
\times \chi(i\omega_m) Z_s(i\omega_m) \Delta_d(i\omega_m, \phi'), \qquad (11)
$$

where

<span id="page-3-1"></span>

FIG. 4. (Color online) Temperature dependence of the upper critical field calculated for  $T_c = 5$  K,  $\lambda_s = 2.48$ ,  $\mu_d^* = 0$ ,  $\Omega_0 = 2$  meV, and  $Y = 1$  meV. (Blue, solid line) Magnetic field **B** along the *c* crystallographic axis and Fermi velocity  $v_F = 0.569 \times 10^5$  m/s; (red, dotted-dashed line) **B** along the  $a$  crystallographic axis and  $v_F = 0.909 \times 10^5$  m/s; (red, dashed line) **B** along *a* and  $v_F = 1.170$  $\times$  10<sup>5</sup> m/s; (red, dotted line) **B** along *a* and  $v_F$ = 0.730  $\times$  10<sup>5</sup> m/s. Experimental data are shown by blue circles  $(B$  along  $c)$  and red squares  $(B \text{ along } a)$ .

$$
\chi(i\omega_m) = (2/\sqrt{\beta}) \int_0^{+\infty} dq \exp(-q^2) \tan^{-1}
$$

$$
\times \left[ \frac{q\sqrt{\beta}}{|\omega_m Z_s(i\omega_m)| + i\mu_B H_{c2} \operatorname{sign}(\omega_m)} \right], \quad (12)
$$

 $\beta = \pi H_{c2} v_F^2 / (2 \Phi_0)$ ,  $v_F$  is the Fermi velocity, and  $\Phi_0$  is the unit of magnetic flux.

Figure [4](#page-3-1) shows the comparison between experimental data<sup>[14](#page-5-4)</sup> and  $\mu_0 H_{c2}(T)$  curves calculated assuming  $\lambda_s = 2.48$  and  $\Omega_0 = 2$  meV. For a field applied along the crystallographic *c* axis, the best fit is obtained for a Fermi velocity  $v_F = 0.569$  $\times$  10<sup>5</sup> m/s. Along the *a* crystallographic direction, the fit is less good and the experimental data are contained within the curves corresponding to  $v_F = 0.730 \times 10^5$  m/s and  $v_F = 1.17$  $\times$  10<sup>5</sup> m/s. The values of the Fermi velocity along different crystallographic directions are connected with cylindrical Fermi surfaces.<sup>14</sup>

The London penetration depth  $\lambda(T)$  has been calculated following the procedure described in Ref. [31](#page-6-0) and the results obtained are shown in Fig. [5.](#page-4-0) For this quantity, no experimental data are yet available.

### **B. Tunneling conductance and energy gap**

The real-frequency-axis formulation of the Eliashberg theory allows one to calculate several physical quantities from the numerical solution of two coupled nonlinear singular integral equations involving a frequency- and temperature-dependent complex gap  $\Delta(\omega, T)$  and a renormalization function  $Z(\omega, T)$ .<sup>[22](#page-5-18)</sup> In the one-band, *d*-wave case one  $has<sup>32,33</sup>$  $has<sup>32,33</sup>$  $has<sup>32,33</sup>$ 

$$
\omega Z_s(\omega) = \omega + \Gamma_N \frac{N_Z(\omega)}{c^2 + N_Z^2(\omega)} + \int_{-\infty}^{+\infty} d\omega' \frac{1}{2\pi} \int_0^{2\pi} \times d\phi' \Lambda(\omega, \omega', \phi, \phi') \text{Real}[N_Z(\omega', \phi')],
$$

<span id="page-4-0"></span>

FIG. 5. Temperature dependence of the London penetration depth  $\lambda(T)$  calculated for  $T_c = 5$  K,  $\lambda_s = 2.48$ ,  $\mu_d^* = 0$ ,  $\Omega_0 = 2$  meV, and  $Y=1$  meV. The curve shows the values of  $\lambda^2(T)$  $(1 - 0.1 \text{ K})/\lambda^2(T)$  as a function of the reduced temperature  $T/T_c$ . The low-temperature behavior is highlighted in the inset, showing  $\Delta\lambda/\lambda_0 = [\lambda(T) - \lambda(T=0.1 \text{ K})]/\lambda(T=0.1 \text{ K})$  versus  $T/T_c$ . The solid line is a linear fit of the results obtained from the numerical solution of the Eliashberg equations.

$$
Z_{s}(\omega)\Delta_{d}(\omega,\phi) = \int_{-\infty}^{+\infty} d\omega' \frac{1}{2\pi} \int_{0}^{2\pi} d\phi' [\Lambda(\omega,\omega',\phi,\phi') - \mu_{d}^{*}(\omega_{c},\phi,\phi')\theta(\omega_{c} - |\omega'|)]
$$
  
×Real[ $N_{\Delta}(\omega',\phi')$ ] (13)

with

$$
\Lambda(\omega, \omega', \phi, \phi') = \frac{1}{2} \int_0^{+\infty} d\Omega \alpha^2 F(\Omega, \phi, \phi')
$$

$$
\times \left[ \frac{\tanh\left(\frac{\omega'}{2T}\right) + \coth\left(\frac{\Omega}{2T}\right)}{\omega' + \Omega - \omega - i\delta} - \frac{\tanh\left(\frac{\omega'}{2T}\right) - \coth\left(\frac{\Omega}{2T}\right)}{\omega' - \Omega - \omega - i\delta} \right], \quad (14)
$$

the superconductive density of states being equal to

$$
(1/2 \pi) \text{Real}[\int_0^{2\pi} d\phi' N_Z(\omega, \phi')],
$$
  

$$
N_Z(\omega, \phi) = \omega / \sqrt{\omega^2 - \Delta^2(\omega, \phi)},
$$

and

$$
N_{\Delta}(\omega, \phi) = \Delta(\omega, \phi) / \sqrt{\omega^2 - \Delta^2(\omega, \phi)}.
$$

The numerical solutions of the above equations, obtained for  $\Omega_0 = 2$  meV,  $Y = 1$  meV, and  $\mu_d^* = 0$ , have been used to calculate the temperature dependence of the energy gap, and the energy dependence of the tunneling differential conductance at different temperatures. The obtained curves are shown in Fig. [6.](#page-4-1) The differential conductance can be obtained from the current-voltage characteristic of a metalinsulator-superconductor tunneling junction, and at zero temperature it coincides with the normalized quasiparticle

<span id="page-4-1"></span>

FIG. 6. (Color online) Energy dependence of the tunneling differential conductance calculated in the superconducting phase of NpPd<sub>5</sub>Al<sub>2</sub> assuming  $T_c = 5$  K,  $\lambda_s = 2.48$ ,  $\mu_d^* = 0$ ,  $\Omega_0 = 2$  meV, and  $Y=1$  meV. Black solid line:  $T=0.25$  K; red dashed line:  $T=2$  K; blue dotted line:  $T=4$  K. The inset shows the temperature dependence of the energy gap calculated for the same set of parameters.

density of states. Point-contact spectroscopy measurements to determine these quantities are planned at ITU.

### **C. Local spin susceptibility and spin-lattice relaxation rate**

Finally, we have calculated the temperature dependence of the local spin susceptibility  $\chi$  and of the nuclear-spinlattice relaxation rate  $T_1^{-1}$  in the superconducting phase of

<span id="page-4-2"></span>

FIG. 7. (Color online) Temperature dependence of (upper panel) the normalized nuclear spin-relaxation rate,  $(T_1 T)^{-1} / (T_1 T)_c^{-1}$ , and (lower panel) the normalized local spin susceptibility,  $\chi(T)/\chi(T_c)$ , calculated in the superconducting phase of  $NpPd<sub>5</sub>Al<sub>2</sub>$ , assuming  $\lambda_s = 2.48$ ,  $\mu_d^* = 0$ ,  $\Omega_0 = 2$  meV,  $Y = 1$  meV, and several different sets of values for the c and  $\Gamma$  parameters: (solid orange line)  $c=0$  and  $\Gamma$ =0.391 meV; (olive dash line)  $c$ =0.5 and  $\Gamma$ =0.4885 meV; (blue dashed-dotted line)  $c=1$  and  $\Gamma=0.782$  meV; (red dotted line)  $c$  $=\infty$  and  $\Gamma/c = 0.391$  meV; (green dashed-dotted-dotted line)  $c = 3$ and  $\Gamma = 3.91$  meV. Experimental data are shown by open circles.

 $NpPd<sub>5</sub>Al<sub>2</sub>$ , and compared the results with the experimental data obtained by Chudo *et al.*<sup>[11](#page-5-11)</sup> from <sup>27</sup>A1 NMR measurements. The data were collected in presence of a magnetic field, reducing the critical temperature to 4 K. This reduction has been simulated in the calculations by introducing an impurity-scattering parameter  $\Gamma$  (Ref. [34](#page-6-3)) leading to the correct value of  $T_c$ . We assumed  $\lambda_s = 2.48$ ,  $\mu_d^* = 0$ ,  $\Omega_0 = 2$  meV,  $Y=1$  meV, and different sets of values for the *c* and  $\Gamma_N$ parameters, going from the unitary limit  $(c=0)$  and  $\overline{\Gamma}_N$  $(1 - 0.391 \text{ meV})$  to the Born limit  $(c = \infty \text{ and } \Gamma_N/c)$  $= 0.391$  meV).<sup>[8](#page-5-6)</sup> The calculated and measured curves, normalized at the values assumed at  $T = T_c$ , are shown in Fig. [7.](#page-4-2) For the local spin susceptibility, a good agreement is obtained in the unitary limit. On the other hand, the experimental behavior of the spin-lattice relaxation rate is better described in the Born limit. Despite this discrepancy, the overall qualitative agreement is as good as can be expected, considering the approximations we used in our simple *d*-wave model.

## **VI. CONCLUSIONS**

The Eliashberg theory of superconductivity has been used to estimate the energy scale  $\Omega_0$  of the electron-boson spectral function in the heavy-fermion, *d*-wave superconductor  $NpPd<sub>5</sub>Al<sub>2</sub>$ . Assuming a spectral function shape typical for antiferromagnetic spin fluctuations and imposing constraints provided by the experimental values for the critical temperature and the low-temperature energy gap, we predict values of  $\Omega_0$  of about 2–2.5 meV, slightly dependent from the strength of the Coulomb pseudopotential. Although the calculations presented here are independent from the particular nature of the Cooper-pair mediating bosons, the observation by NMR experiments of spin fluctuations with a maximum spectral weight at energies close to the predicted value of  $\Omega_0$ provide further support to the hypothesis of magnetically mediated superconductivity in  $NpPd<sub>5</sub>Al<sub>2</sub>$ . Inelastic neutronscattering experiments would be interesting to confirm the proposed scenario.

Calculations provide values for the electron-boson coupling constant  $\lambda_s$  of about 2.5, confirming that NpPd<sub>5</sub>Al<sub>2</sub> is a strong-coupling superconductor. Finally, we used the solutions of the Eliashberg equations to calculate the temperature dependence of several physical quantities, such as the upper critical field, the London penetration depth, the local spin susceptibility, the nuclear-spin-lattice relaxation rate, and the energy gap, as well as the energy dependence of the tunneling differential conductance at various temperatures. The calculated curves compare favorably with available experimental data and provide predictions for future experimental work.

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